CURBSIDE PARKING TIME LIMITS*

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Abstract

This paper investigates the economics of curbside parking time limits. It argues that curbside parking time limits provide a way to subsidize short-term parking without generating cruising for parking. The paper develops the argument in the context of the integrated model of downtown parking and traffic congestion presented in Arnott and Rowse (2009), extended to incorporate heterogeneity in value of time and parking duration.

Keywords: parking, curbside parking, time limits, parking garages, parking policy

JEL Classification: R41, L91, H71, H21, D62

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1. Introduction

A debate has recently emerged in both the academic and policy literatures on the pricing of curbside parking downtown. Current practice has been strongly influenced by downtown merchant associations that believe that downtown parking should be subsidized to make downtown shopping areas competitive with downtown shopping centers, where parking is typically provided free. The garage parking of downtown shoppers is subsidized through "validation"[1] of garage parking, curbside parking through its "underpricing". The opponents of underpricing curbside parking present two types of arguments against the practice. The first-best argument is that curbside parking should be "cashed out" -- priced to clear the market. Underpriced curbside parking is rationed via cruising for parking. The deadweight loss associated with this rationing mechanism is not only the value of time lost by cars cruising for parking but also the value of time lost due to the increased traffic congestion that cruising for parking causes. Pricing curbside parking to clear the market eliminates this efficiency and ensures that curbside parking spots go to those who value them the most. The second-best argument is that, even when subsidizing curbside parking is desirable, attempting to achieve this goal by underpricing curbside parking is dysfunctional since it actually causes the price of a downtown shopping trip to *increase.*

[1] An individual parks in a parking garage and shops at a store. At the checkout, he presents his parking stub to the store clerk, who validates it. Upon exiting the garage, the individual presents his validated parking stub to the parking attendant, who waives part or all of the parking fee. The merchant then reimburses the garage for part or all of the waived portion of the parking fee.

The arguments against underpricing curbside parking have been developed in the context of models with identical individuals. This paper makes the simple point that these arguments are significantly weakened when account is taken of driver heterogeneity; in particular, when drivers differ in visit duration, curbside parking time limits can be used to eliminate cruising for parking, so that supplementing the underpricing of curbside parking with curbside parking time limits can be an effective way of subsidizing curbside parking. This paper develops this point, and more generally investigates the economics of curbside parking time limits, by extending Arnott and Rowse's (2009) integrated model of downtown curbside parking, garage parking, and traffic congestion to include drivers who differ in both value of time and visit duration.

Section 2 of the paper reviews the relevant literature. Section 3 extends the simple model of Arnott and Rowse (2009), to treat first the case where individuals differ in terms of only the value of time, second the case where individuals differ in terms of only parking duration, and third the case of central interest, where individuals differ in terms of both value of time and parking duration. Section 4 then explores quantitative aspects of the economics of downtown parking policy through an extended numerical example, calibrated to simulate a medium-size, auto-oriented US city. Section 5 concludes.

2. Literature Review

Early work on the economics of parking argued that parking, like any other commodity, should be priced at its social opportunity cost (Vickrey, 1954; Roth, 1965). Over the next quarter century, parking was largely ignored by economists, in modal choice studies being treated simply as a fixed cost added to an auto trip. Donald Shoup has done much to stimulate recent interest in the subject. In the 1990's he championed cashing out employer-provided parking[2](Small and Verhoef (2007) estimate that in the US employees pay on average only 5% of the costs associated with employer-provided parking) and over the last decade cashing out shopping center and curbside parking. His extensive research and policy advocacy on the subject is synthesized in Shoup (2005). Three noteworthy academic papers were written on the economics of parking during the 1990's. Glazer and Niskanen (1992) pointed out possible perverse effects from per-unit-time curbside parking fees when auto congestion is unpriced and parking duration a choice variable; Arnott, de Palma, and Lindsey (1992) extended the Vickrey bottleneck model (1969) to analyze the temporal-spatial equilibrium of curbside parking when all drivers have a common destination and desired arrival time (Anderson and de Palma, 2004, later extended the model), such as for a special event or the morning commute; and Arnott and Rowse (1999) examined the steady-state equilibria of cars cruising for parking on a circle when parking is unsaturated.

The body of research most relevant to this paper, which considers some combination of curbside parking, garage parking, and traffic congestion with identical individuals, has been developing over the last decade. Arnott and Inci (2006) elaborated a model first presented in Arnott, Rave, and Schöb (2005, Ch. 2) in which all parking is curbside and cars cruising for parking contribute to traffic congestion. If the curbside parking capacity

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[2] The term "cashing out" is used somewhat ambiguously. In the context of shopping center and curbside parking, it means simply rationing by price. In the context of employer-provided parking, it may mean either this or leveling the playing field between auto and mass transit commuting by giving employees a cash budget for commuting.

constraint binds, the equilibrium full price of a trip is determined by the intersection of the trip demand curve and the curbside parking capacity constraint. If also curbside parking is priced below its social opportunity cost, there is excess demand for curbside parking, which is rationed via cruising for parking. In particular, the stock of cars cruising for parking adjusts so as to bring the full trip price (the parking fee plus in-transit travel time costs plus cruising-for-parking time costs) up to its equilibrium level. In this model, raising the meter rate has no effect on the full price of a trip, and simply converts travel time costs dollar for dollar into meter revenue. Thus, revenue is raised from parking meters with no burden at all.

Calthrop (2001) was the first to consider a model with both curbside and garage parking. He recognized that, if curbside parking is priced below garage parking, then some dissipative activity occurs to equalize the full prices of curbside and garage parking. The dissipative activity he considered was queuing. Arnott, Rave, and Schöb (2005) presented a model in which the dissipative active is cruising for parking[3], with the cars cruising for parking contributing to traffic congestion, which was subsequently elaborated by Arnott and Rowse (2009). Suppose that the garage parking fee is independent of the meter rate, at least as long as curbside parking is priced below the marginal cost of a garage parking space[4]. Now consider the effects of raising the meter

[3] Shoup (2005, 2006) developed a similar model, but did not consider the interaction between cars cruising for parking and traffic congestion.

[4] Arnott and Rowse (2009) considered two models. In their "simple" model, garage parking is supplied at constant marginal cost. In their "central" model, garages are discretely spaced, and the garage parking fee is determined by spatial competition between parking garages.

rate slightly. Since this does not affect the full price of garage parking, neither does it affect the full price of curbside parking. Thus, the rise in the meter rate converts cruising-for-parking time costs dollar for dollar into meter revenue. Since cruising-for-parking time costs are reduced via a reduction in the stock of cars cruising for parking, there is the added benefit that traffic congestion is reduced, which benefits everyone. Thus, the rise in the meter rate not only raises revenue but also increases consumer surplus; the revenue raised from parking meters generates negative burden – a double dividend result. And if account is taken that the marginal cost of public funds exceeds unity, a triple dividend is achieved.

To our knowledge, no paper in the economics literature on parking considers curbside parking time limits. Thus, the impression left by the current literature is that underpricing curbside parking is dysfunctional. Not only does it cause needless inefficiency and cause local government to forgo a potentially lucrative source of revenue, but by increasing traffic congestion it *increases* the cost of a shopping trip downtown – hence having the opposite effect to that intended.

3. Equilibria: Different Model Variants

We start this section by reviewing the simple model of Arnott and Rowse (2009) in which everyone has the same value of time and parks for the same exogenous period of time, and in which therefore parking time limits play no role. We then extend the model

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Calthrop (2001) considered a model in which garage parking is supplied by a monopolist, and in which the curbside parking fee and the garage parking fee are determined as the outcome of a price game between the municipal authority in charge of curbside parking and the monopoly garage owner.

to incorporate heterogeneity in the value of time. Since everyone continues to park for the same exogenous period of time, parking limits still play no role. This case is not, however, without interest since individuals sort themselves between curbside and garage parking on the basis of their value of time. We next extend the model to incorporate heterogeneity in parking duration but not the value of time, and examine the effects of curbside parking time limits. We finally extend the model to consider heterogeneity in both parking duration and the value of time. Adding heterogeneity in the value of time to heterogeneity in parking duration does not fundamentally alter the role of parking time limits, but permits consideration of the distributional effects of alternative policies.

3.*1* *Setting the stage: identical individuals*

To set the stage, we review the simple model of Arnott and Rowse (2009). In that model, there is an inelastic and time-invariant flow of trips to each unit area of downtown, D. Demand is sufficiently high that garage parking is needed to supplement curbside parking but not so high that it cannot be accommodated by the street system. Each trip entails a car driver traveling a distance δ over the downtown area to his destination, visiting there for duration λ, and then exiting. Drivers are identical. Each driver has the choice of curbside or garage parking. If he parks curbside, he may have to cruise around the block until a parking space opens up, while parking garage spaces are available immediately. Curbside parking is managed by the government, garage parking by the private sector. The government decides on the curbside meter rate per unit time, f, and the number of curbside parking spaces per unit area, P. Garage parking is constructed at an amortized constant unit cost of c per unit time, and, like curbside parking, is provided continuously over space. Since everyone therefore parks at his destination, parking duration equals visit duration. Competition between garage operators forces the garage parking fee down to unit cost. The curbside meter rate is set below the garage parking fee. As a result, curbside parking is saturated (as soon as a parking space is vacated, it is filled by a car that is cruising for parking), and the stock of cars cruising for parking adjusts to equalize the full prices of curbside and garage parking. Travel is subject to flow congestion; in particular, travel time per unit distance, t, is increasing (and convex) in the stock of cars in transit per unit area, T, the stock of cars cruising for parking per unit area, C, and the stock of curbside parking spaces per unit area: $t = t(T,C,P)$.

Equilibrium solves two equations in the two unknowns, T and C. The first equilibrium condition, the *steady-state condition,* is that in steady state the flow of cars entering the in-transit pool per unit area equals the flow of cars exiting it. The flow of cars entering it is D. The flow of cars exiting it equals the stock of cars in transit divided by the length of time each car spends in the pool, which equals its distance traveled times travel time per mile:

$$D = \frac{T}{\delta t(T,C,P)}. \qquad (1)$$

The *cruising-for-parking equilibrium condition* states that the stock of cars cruising for parking is such that the full price of garage parking equals the full price of curbside parking. The full price of garage parking is simply $c\lambda$. The full price of curbside parking has two components, the curbside parking payment, $f\lambda$, and the expected time cost of cruising for parking. The expected time cost of cruising for parking equals the expected time cruising for parking times the value of time, ρ. The expected time cruising for parking equals the reciprocal of the Poisson rate at which a driver encounters a curbside parking spot being vacated. Since curbside parking spots are vacated at the rate $\frac{P}{\lambda}$ per unit area, and since there are *C* cars cruising for parking per unit area, the rate at which a driver encounters a parking spot being vacated is $\frac{P/\lambda}{C}$. Thus,

$$c\lambda = f\lambda + \frac{\rho C\lambda}{P}, \tag{2}$$

which can be solved to determine the stock of cars cruising for parking:

$$C = \frac{(c - f)P}{\rho}; \tag{3}$$

the equilibrium stock of cars cruising for parking is linearly proportional to the price differential between garage and curbside parking, to the stock of curbside parking spaces, and to the reciprocal of the value of time. With *C* so determined, (1) can be solved for *T*. There are two roots. One can be ruled out on the basis of a stability argument, which leaves a unique equilibrium with the property that the stock of cars in transit is increasing in the stock of cars cruising for parking, so that (1) can be written as $T = T(C;P)$ with $\frac{dT}{dC} > 0$. Having solved for the equilibrium *C* and *T*, all other variables of interest can be solved for.

The model with identical individuals is inappropriate for examining curbside parking time limits. If the time limit is greater than the common parking duration, it has no effect, and if it is less, then it forces everyone to garage park.

3.2 Individuals have the same visit length but differ in the value of time

This case is not of great interest in the context of this paper since, when individuals have the same, fixed visit duration, there is no role for curbside parking time limits. We record the equilibrium conditions as an intermediate step. Eq. (1), the steady-state equilibrium condition, continues to apply. Those individuals with a low value of time choose to park curbside rather than in a parking garage since the money savings from doing so more than offset the cruising-for-parking time costs they incur when curbside parking. Contrarily, those individuals with a high value of time are willing to pay the premium to garage park so as to avoid cruising-for-parking time costs. At the boundary between these two groups is a *marginal parker*, who is indifferent between parking on the curb or in a garage. $\tilde{\rho}$, the marginal parker's value of time, is determined by the condition

$$D\lambda J(\tilde{\rho}) = P \,, \tag{4}$$

where $J(.)$ is the cumulative distribution function of ρ. $J(\tilde{\rho})$ is the proportion of individuals who have a value of time less than that of the marginal parker, and is therefore the proportion of individuals who choose to park on street. $DJ(\tilde{\rho})$ is therefore the entry rate of individuals into curbside parking, so that the number of occupied curbside parking spaces is $D\lambda J(\tilde{\rho})$, which, since curbside parking is saturated, equals the number of curbside parking spaces. The cruising-for-parking equilibrium condition applies to the marginal parker. Thus,

$$c\lambda = f\lambda + \frac{\tilde{\rho}C\lambda}{P}. \tag{5}$$

3.3 Individuals have the same value of time but differ in visit length.

In this case, there is a role for curbside parking time limits. Eq. (1) continues to be the steady-state equilibrium condition. But the cruising-for-parking equilibrium condition changes again. First, the marginal parker is now identified by a particular visit duration. Those parkers with shorter visit durations choose to park in a garage; the parking fee savings they would derive from parking curbside rather than in a garage are not enough to compensate for the fixed cruising-for-parking time costs associated with curbside parking. Those parkers with longer visit durations choose to park curbside. Second, account needs to be taken that the curbside parking turnover rate is determined by the average visit duration of curbside parkers and not the visit duration of the marginal curbside parker.

Let $h(\lambda)$ denote the probability density function of visit duration, $H(\lambda)$ the corresponding cumulative distribution function, $\Lambda(\lambda')$ the mean visit duration conditional on λ being greater than λ', and $\tilde{\lambda}$ the visit duration of the marginal parker. To avoid some complications that are of little economic interest, we shall assume that the minimum visit duration is zero. The analog to (4) for the previous case is

$$D\int_{\tilde{\lambda}}^{\infty} \lambda h(\lambda)d\lambda = P, \tag{6a}$$

which can be rewritten as

$$D\Lambda(\tilde{\lambda})\left(1 - H(\tilde{\lambda})\right) = P\,. \tag{6b}$$

The cruising-for-parking equilibrium condition is

$$(c - f)\tilde{\lambda} = \frac{\rho C\Lambda(\tilde{\lambda})}{P}\,. \tag{7a}$$

The left-hand side of the equation is the money saving to the marginal parker from parking curbside rather than in a garage. In equilibrium, this equals his expected value of time lost cruising for parking. The curbside parking turnover rate equals the stock of curbside parking spaces divided by the average time parked for curbside parkers, which equals the average time parked for individuals with visit duration exceeding $\tilde{\lambda}$. The Poisson arrival rate of opportunities to park curbside for a particular driver cruising for parking is therefore $\frac{P/\Lambda(\tilde{\lambda})}{C}$, and the expected time lost cruising for parking is the reciprocal of this. Note from (6b) that the curbside parking turnover rate, $\frac{P}{\Lambda(\tilde{\lambda})}$, equals $D\big(1 - H(\tilde{\lambda})\big)$, which is the entry rate into curbside parking. Having solved the equilibrium *C* and *T*, all other variables of interest can be solved. . Eq. (7a) can be rewritten as

$$C = \frac{P(c-f)\tilde{\lambda}}{\rho\Lambda(\tilde{\lambda})}, \qquad \text{(7b)}$$

which is the same as the corresponding expression for the stock of cars cruising for parking with identical individuals, (3), except that it is multiplied by the ratio of the visit duration of the marginal parker to the average visit duration of curbside parkers. As we shall see, curbside parking limits can be effective in reducing the stock of cars cruising for parking through their effect on this ratio.

We are now in a position to introduce curbside parking time limits. Let τ denote the legislated curbside parking time maximum[5]. The parking time limit could be so severe that curbside parking is not saturated. For parking time limits that are not as severe as this, curbside parking remains saturated and is occupied by parkers with visit durations

[5] We assume that enforcement is perfect and costless, so that an individual cannot park for longer than the allowable time by returning to “feed the meter”.

between $\tilde{\lambda}$ and τ, where $\tilde{\lambda}$, the visit duration of the marginal parker, is determined by the condition that

$$D\int_{\tilde{\lambda}}^{\tau} \lambda h(\lambda) d\lambda = P, \tag{8a}$$

which gives an implicit equation for $\tilde{\lambda}$ as a function of τ, $\tilde{\lambda}(\tau)$. Differentiating (8a) with respect to τ yields

$$\frac{d\tilde{\lambda}(\tau)}{d\tau} = \frac{\tau h(\tau)}{\tilde{\lambda} h(\tilde{\lambda}(\tau))} > 0\ ; \tag{9}$$

thus, lowering the curbside parking time limit reduces the visit duration of the marginal parker. With some abuse of notation, let $\Lambda(\tilde{\lambda}(\tau), \tau)$ denote the average visit duration of curbside parkers with a visit duration between $\tilde{\lambda}(\tau)$ and the curbside parking time limit τ. Then (8a) can be rewritten as

$$D\Lambda\big(\tilde{\lambda}(\tau), \tau\big)\Big(H(\tau) - H\big(\tilde{\lambda}(\tau)\big)\Big) = P\ . \tag{8b}$$

The cruising-for-parking equilibrium condition applies to the marginal parker. Thus,

$$(c - f)\tilde{\lambda}(\tau) = \frac{\rho C \Lambda\big(\tilde{\lambda}(\tau), \tau\big)}{P}\ , \tag{10a}$$

which can be can be rewritten as

$$C = \frac{(c - f)P\tilde{\lambda}(\tau)}{\rho\Lambda(\tilde{\lambda}(\tau), \tau)}\ . \tag{10b}$$

Eqs. (10a) and (10b) have the same interpretations as (7a) and (7b). In the limit, as $\tilde{\lambda}(\tau)$ is reduced to zero, cruising for parking is eliminated. Later we shall use the result that

$$sgn\left(\frac{dC}{d\tau}\right) = \tau[H(\tau) - H(\tilde{\lambda})] - (\tau - \tilde{\lambda})h(\tilde{\lambda})\tilde{\lambda}, \tag{11}$$

which is obtained from total differentiation of (8b) and (10b).

Curbside parking time limits hurt drivers with long visit durations since they are forced to pay the higher garage parking fee throughout their long visits. Thus, determining the optimal curbside parking time limit entails distributional considerations. We shall adopt the particular distributional assumption that a dollar is equally valued to whomever it is given, in which case the optimal curbside parking time limit minimizes overall resource costs. While we will not consider alternative welfare functions, later in our numerical work we shall calculate and comment on the distributional effects of various policies.

There are three components to resource costs: garage parking costs, cruising-for-parking time costs, and in-transit travel time costs. As long as curbside parking remains saturated when the curbside parking time limit is applied, which we have assumed, the same number of garage parking spaces are used, so that garage parking costs are independent of the parking time limit. Cruising-for-parking time costs are minimized when the stock of cars cruising for parking is zero. And under the stability criterion we employed to select the equilibrium, in-transit travel time costs are minimized when the stock of cars cruising for parking is zero. The optimal curbside parking time limit therefore eliminates cruising for parking without curbside parking becoming unsaturated. From (11), this is achieved by setting τ such that the $\tilde{\lambda}$ solving (9) is zero. Adjusting the curbside parking time limit affects the identity of the marginal parker. We would like the marginal parker to have a zero visit length. Since his money savings from curbside rather than garage parking are then zero, to be indifferent between curbside and garage parking requires that his cruising-for-parking time costs be zero, which in turn requires that the stock of cars cruising for parking be zero.

In the introduction, we gave a different rationale for curbside parking time limits. We argued there that time limits provide a means of eliminating the excess demand created by subsidizing curbside parking. This is achieved with a maximum curbside parking time limit set such that $\tilde{\lambda} = 0$ solves (9). Thus, the two rationales are consistent with one another[6] [7].

3.4 Individuals differ in both visit length and value of time

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[6] When the minimum visit duration, $\lambda_{min,}$ is strictly positive, under both explanations there is an element of indeterminacy. Any stock of cars cruising for parking between zero and $\frac{(c-f)\lambda_{min}\psi^e}{\rho}$, where ψ^e is the equilibrium turnover rate, is consistent with equilibrium. In the marginal parker explanation, the requirement that the parker with the minimum parking duration weakly prefer curbside parking to garage parking is consistent with the indicated range of *C*, and in the excess demand explanation the quantity of curbside parking demanded equals the quantity supplied over the indicated range.

[7] If the government decides on the curbside parking fee, the proportion of curbside to allocate to parking, and curbside parking time limits independently, it is likely to end up with either unused curbside parking spaces or cruising for parking, both of which are wasteful. It can set any two of the instruments independently, but then should use the third to make sure that at the same time all curbside parking is utilized and there is no cruising for parking. We demonstrated this point by showing how curbside parking time limits can be set to satisfy this criterion, taking as given the meter rate and the stock of curbside parking spaces. But the same point applies if any other pair of policy instruments is determined first, and the third adjusted to satisfy the criterion. For example, the local government might decide that it wishes to provide free parking to all drivers who park for less than an hour. To achieve this without either excess supply or excess demand, it would need to set *P* appropriately.

In this general case, there is a continuum of marginal parkers. In the absence of a curbside parking time limit, for every value of time there is a visit duration above which individuals choose to park on street. Refer to this function as $\varepsilon(\rho)$. And let $g(\rho,\lambda)$ denote the joint p.d.f. of the value of time and visit duration. Then the analog to (4) is

$$D\int_{0}^{\infty}\left[\int_{\varepsilon(\rho)}^{\infty}\lambda g(\rho,\lambda)d\lambda\right]d\rho = P. \tag{12}$$

The curbside turnover rate equals the entry rate into curbside parking, which equals

$$\psi = D\int_{0}^{\infty}\left[\int_{\varepsilon(\rho)}^{\infty} g(\rho,\lambda)d\lambda\right]d\rho\,. \tag{13}$$

Eqs. (12) and (13) together imply that the curbside turnover rate equals the stock of curbside parking spaces divided by the average time parked of curbside parkers. The analog of (5) for every ρ is

$$(c-f)\varepsilon(\rho) = \frac{\rho C}{\psi}\,. \tag{14}$$

The left-hand side is the money saving from curbside rather than garage parking for the marginal parker with value of time ρ. In equilibrium this must be offset by expected cruising-for-parking time cost[8] [9].

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[8] We may view the causality as follows. For a given ρ, (14) determines $\varepsilon(\rho)$ as a function of C and ψ, so that we may write $\varepsilon(\rho;\ C,\psi) = \frac{\rho C}{\psi(c-f)}$. Substituting this into (12) and (13) gives two equations in the two unknowns, C and ψ. Noting that C and ψ enter (12) only as $\frac{C}{\psi}$, solve this equation for $\frac{C}{\psi}$. There is a unique solution since the LHS of (12) is continuous and monotonically decreasing in $\frac{C}{\psi}$ while the RHS is a constant that lies between the maximum and minimum values of the LHS. Having solved for $\frac{C}{\psi}$, the equilibrium ψ may be solved from (13), and then the equilibrium C. The equilibrium T is obtained from (1), and then all other variables of interest may be determined

The equilibrium with a curbside parking time limit is solved in the same way[10], except that the limits on the integrals in (12) and (13) are modified to incorporate the curbside parking time limit. As in the previous case, the optimal time limit (again defined to be that which minimizes resource costs) eliminates cruising for parking, with parking remaining saturated. Curbside parking time limits play the same role as in the previous case.

3.5 Social optima and their decentralization

We earlier defined an allocation to be socially optimal if it minimizes aggregate resource costs. This definition assumes that society considers only efficiency and that trip demand is fixed[11].

In the model, a social planner has only two sets of choices, how much curbside and garage capacity to provide, and how to allocate the heterogeneous individuals between curbside and garage parking. Curbside should be allocated to parking to the point where the social cost of an extra curbside parking space, the increase in aggregate in-transit

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[9] The Appendix examines the effects of an increase in the amount of curbside parking, which will be discussed in explaining the results of the numerical example.

[10] If the time limit is set too short, then on-street parking is not fully occupied and (12) cannot be satisfied. How the equilibrium is solved in this case is explained in fn.24.

[11] Our assertions that a set of policies decentralizes the social optimum applies only in the context of our model. The decentralization mechanisms might not work well in reality since they might provide inappropriate incentives on margins that we have ignored. For example, we have taken visit duration as fixed. A set of policies that decentralizes the social optimum in our model might not provide the appropriate incentives with respect to visit duration.

travel cost resulting from the increased congestion it causes, equals the social benefit, the resource saving from providing one less garage space. Garage parking should be provided to accommodate the residual demand. There should be no cruising for parking, and with no cruising for parking, curbside and garage parking spaces are perfect substitutes, so that how individuals are allocated between them has no effect on social cost. This social optimum can be decentralized simply by having the government choose the efficient amount of curbside parking, price it at its social opportunity cost, and leave the market to determine garage capacity and pricing. The full social optimum can also be decentralized by having the government choose the efficient amount of curbside parking, price it below its social opportunity cost, impose the optimal curbside time limit, and leave the market to determine garage capacity and pricing.

We could consider a range of second-best problems. We might, for example, wish to treat explicitly the distortions that are asserted to justify the subsidization of curbside parking downtown. But we shall consider only two second-best issues. First, what is the second-best optimum when the amount of curbside parking is inefficient and cannot be altered, and how can this second-best optimum be decentralized? Second, what is the second-best optimum when curbside parking is underpriced and cannot be altered, and how can this second-best optimum be decentralized?

Suppose that the amount of curbside parking is fixed at an inefficiently high level[12]. We have assumed that congestion depends on the number of designated curbside parking spaces, as distinct from the number of curbside parking spaces actually used. Under this assumption[13], the second best is to fully utilize all the designated curbside parking spaces with no cruising for parking. Cruising for parking costs can be eliminated either by pricing curbside parking at its social opportunity cost, c, or by subsidizing curbside parking and using a parking time limit to eliminate cruising for parking. Suppose alternatively that the amount of curbside parking is inefficiently low. Second-best efficiency again entails eliminating cruising for parking, while keeping curbside parking saturated, which can be achieved either by setting the curbside parking fee equal to the garage parking fee or setting the curbside parking fee below the garage parking fee and setting the curbside parking time limit so as to eliminate cruising for parking without curbside parking becoming unsaturated.

Suppose that the curbside meter rate is set below its social opportunity cost, evaluated at the social optimum, and cannot be altered. In the context of our model, the social optimum can be decentralized by setting the amount of curbside parking at its first-best level and then subsidizing garage parking by the same amount as curbside parking. It can also be decentralized by setting the amount of curbside parking at its first-best level and

[12] Recall that we have assumed that, even if all curbside is allocated to parking, it is insufficient to meet the total demand for parking.

[13] An alternative assumption is that congestion depends on the number of curbside parking spaces actually used. Under this alternative assumption, the first best can be decentralized by setting the curbside parking time limit such that the first-best efficient number of curbside parking spaces is used, and then setting the curbside meter rate equal to c.

then imposing the optimal curbside parking time limit. If neither of these policies instruments is feasible, the second-best policy is to reduce the amount of curbside parking below the first-best level. Doing so decreases cruising-for-parking costs since a smaller fraction of the population parks curbside and therefore cruises for parking, and hence reduces in-transit travel costs, but increases garage parking costs.

4. An Extended Numerical Example

Arnott and Rowse (2009) presented an extended numerical example for a model similar to the one presented here, except that it treated the case of homogeneous drivers and assumed that garage parking is subject to a form of economies of scale that confers market power on garage operators, leading to the garage parking fee being determined as the outcome of a game between garage operators. Except for differences deriving from these differences in model specification, we adopt the parameters and functional forms employed in that paper.

The following base case parameters are drawn from that paper[14]. The units of measurement are hours for time, miles for distance, and dollars for value.

$$\delta = 2.0 \qquad f = 1.0 \qquad P = 3712 \qquad D = 7424$$

The in-transit travel distance is 2.0 mls; the curbside parking fee is $1.00/hr; the number of curbside parking spaces is 3712 per ml^2, which corresponds to curbside parking on one

[14] The choice of parameter values is explained in Arnott and Rowse (2009). Suffice it to say here that they are chosen to be representative of medium-sized cities in high-income countries, such as Winnipeg, Perth, San Diego, Sacramento, and Phoenix.

side of every street; and the entry rate into the downtown area is 7424 per ml^2-hr. We also assume that the unit cost of a garage parking space, c, is[15] \$3.00/hr.

The form of the congestion function employed is

$$t = \frac{t_0}{\left(1 - \frac{V}{V_j}\right)}, with\ V = T + \theta C\ and\ V_j = \Omega\left(1 - \frac{P}{P_{max}}\right). \tag{15}$$

Travel time per mile, t, is an increasing function of the effective density of traffic, V. When effective density is zero, travel time per mile is free-flow travel time, t_0; and in the limit as effective density approaches jam density, V_j, travel time per mile is infinite. θ is the number of in-transit car-equivalents that a car cruising for parking contributes to congestion; thus, effective density is the density of traffic in terms of in-transit car equivalents. The jam density is negative linearly related to the proportion of street space allocated to curbside parking; if no street space is allocated to curbside parking, jam density is Ω, and if P_{max} street space is allocated to curbside parking, jam density is zero. The four parameters of the congestion function are taken from Arnott and Rowse (2009):

$t_0 = 0.05$ $\quad\Omega = 5932.38$ $\quad P_{max} = 11136$ $\quad\theta = 1.5$

Free-flow travel time is 0.05 hrs, corresponding to a free-flow travel speed of 20 mph; effective jam density with no on-street parking is 5932.38 in-transit car equivalents per ml^2; if all street space is allocated to curbside parking, the curbside parking density would

[15] Arnott and Rowse (2009) assumed a parameterized garage cost function that accounts for the economies of scale that give rise to the discrete spacing of parking garages and garage operators' market power. Here, instead, to abstract from the complications caused by spatial competition between parking garages, which are not of central interest in the context of this paper, we assume that garage parking is uniformly provided over space at constant unit cost.

be 11136 parking spaces per ml^2; and a car cruising for parking contributes 1.5 times as much to congestion as a car in transit.

Individuals differ in terms of both the value of time and visit duration. We assume the population probability density function to be

$$g(\rho,\lambda) = j(\rho)h(\lambda) = \left\{\frac{e^{\left[\frac{-(ln\,\rho-\mu)^2}{2\sigma^2}\right]}}{x\sigma(2\pi)^{\frac{1}{2}}}\right\}\{\gamma e^{-\gamma\lambda}\}. \tag{16}$$

The marginal probability densities of value of time and visit duration are assumed to be independent. Thus, the joint probability density of the value of time and visit duration is the product of the marginal probability densities. The marginal probability density of the value of time is assumed to be lognormally distributed[16], with the μ and σ in (16) being the mean and standard deviation of $ln\ \rho$, and visit duration is assumed to be negative exponential distributed with mean $\frac{1}{\gamma}$. The mean and the standard deviation of the value of time are calculated on the basis of estimates of the 25th and 75th percentile values of time reported in Small, Winston, and Yan (2005)[17]. And mean visit duration is assumed to be 2.0 hrs[18]. The parameter values so calculated are

[16] The empirical literature provides strong support for the assumption that the value of time is proportional to the wage rate, and for the assumption that wages are lognormally distributed in the population.

[17] Small, Winston, and Yan (2005) estimate the 25th and 75th percentile values of time by observing the distribution of the revealed (and stated) marginal rates of substitution between time and money in individuals' choices of whether to travel on the tolled or untolled lanes of a section of State Route 91 in Orange County. On the assumption that the value of time is lognormally distributed, we calculated the mean and the standard deviation of the lognormal distribution consistent with their estimates.

[18] This assumption is rather arbitrary. In our model, visit length equals parking duration. The distribution of parking duration varies considerably over locations. We assumed that the mean parking duration is 2.0 hrs to obtain results consistent with Arnott and Rowse

$\mu = 3.130337 \qquad \sigma = 2.136014 \qquad \gamma = 0.5.$

The corresponding mean and standard deviation for the value of time are 22.881653 and 8.4656523, respectively. With an average parking duration (= visit duration) of 2.0 hrs, an arrival rate into the downtown area of 7424 cars per ml^2-hr, and 3712 curbside parking spaces per ml^2, there are three times as many garage parking spaces as curbside parking spaces.

Table 1 presents the numerical results. Each column corresponds to a different exercise. The results of the base case equilibrium, in which the curbside meter rate is $1.00 /hr and curbside parking is on one side of the street (so that $P = 3712$), are given in column 3. Column 1 describes the social optimum, with the amount of curbside parking set at its base case level. Column 2 corresponds to the social optimum, with the amount of curbside parking optimized (and hence the first-best level of curbside parking). Columns 3 through 7 record various equilibria, each corresponding to a different policy. The meter rate remains set at $1.00/hr throughout. Column 4 displays the equilibrium when the amount of curbside parking is (second-best) optimized. Thus, comparison of columns 3 and 4 indicates how the equilibrium changes when curbside parking is optimized rather than set at its base case level. Column 5 applies to the equilibrium when the base case parking policies are supplemented by a curbside parking time limit of 2.0 hrs. Column 6 describes the equilibrium when the base case parking policies are supplemented by the optimal curbside parking time limit, and column 7 the equilibrium when both the amount of curbside parking and the curbside parking time limit are optimized. Column 8

(2009) which assumes a common parking duration of 2.0 hrs, and that parking duration is negative exponential distributed.

Table 1: The effects of curbside parking limits with individuals who differ in value of time and visit duration

Column	1	2	3	4	5	6	7	8
	SO	$SO(P^*)$	E	$E(P^{**})$	$E(\tau = 2)$	$E(\tau = \tau^{**})$	$E(P^*, \tau^*)$	$E(IdInds)$[4]
	P = 3712		P = 3712		P = 3712	P = 3712		P = 3712
			f = 1	f = 1	f = 1	f = 1	f = 1	f = 1
τ						1.923	2.246	
P		4594		1044			4594	
v	14.99	13.85	10.67	15.91	13.73	14.99	13.85	10.12
C	0	0	302.14	110.61	105.36	0	0	342.45
$\frac{C}{T+C}$	0	0	0.178	0.106	0.089	0	0	0.181
ψ	4585.0	5008.9	575.5	119.4	3543.9	4585.0	5008.9	1856.0
$\frac{GC}{D}$	4.500	4.144	4.500	5.578	4.500	4.500	4.144	4.500
$\frac{TT}{D}$	3.053	3.305	4.288	2.876	3.332	3.053	3.305	4.523
$\frac{CP}{D}$	0	0	0.690	0.217	0.312	0	0	1.000
$\frac{RC}{D}$	7.553	7.449	9.478	8.671	8.144	7.553	7.449	10.023
$\chi(10)$[2]			3.560	6.282	0.202	0	0	
$\chi(50)$			5.633	9.941	0.319	0	0	
$\chi(90)$			8.915	15.732	0.505	0	0	
$\overline{F}$			9.978	8.812	8.644	8.053	8.068	10.523
$F(10, 10)$			3.173	2.337	2.589	2.020	2.169	
$F(10, 50)$[3]			6.700	5.863	3.764	3.195	3.345	
$F(10, 90)$			14.266	13.429	15.790	15.625	15.774	
$F(50, 10)$			4.654	3.330	3.757	3.074	3.310	
$F(50, 50)$			8.180	6.856	5.150	4.250	4.486	
$F(50, 90)$			17.837	16.513	16.941	16.678	16.915	
$F(90, 10)$			6.996	4.901	5.578	4.741	5.116	
$F(90, 50)$			10.523	8.428	7.342	5.917	6.292	
$F(90, 90)$			20.179	18.084	18.761	18.346	18.721	

Notes:
1. The unit of distance is a mile, of time is an hour, and of value is a dollar.
2. $\chi(10)$ denotes the visit duration of the marginal parker with the 10th percentile value of time.
3. $F(10, 50)$ denotes the full trip price for the parker with the 10th percentile value of time and the median (50th percentile) visit duration. For each visit duration, the 10th, 50th, and 90th percentiles of the values of time are 13.561, 21.460 and 33.961, respectively. For each value of time, the 10th, 50th, and 90th percentiles of visit duration are 0.211, 1.386, and 4.605, respectively.
4. Individuals are identical with visit duration 2.0 (the mean visit duration of the other cases) and value of time 22.882 (the mean value of time for the other cases). The values for χ and F are left blank. Since individuals are identical, all are indifferent in equilibrium between curbside and garage parking and all face the same full trip price.

indicates the base case equilibrium when individuals are identical.

We start with Column 1, which reports on the properties of the social optimum when the amount of curbside parking is set at the base case level (the base case social optimum). Since there is no cruising for parking, the resource costs associated with parking are independent of the allocation of individuals between curbside and garage parking. *CP* denotes aggregate cruising-for-parking time costs per ml^2-hr, and *D* the inflow rate per ml^2-hr, so that $\frac{CP}{D}$ gives mean cruising-for-parking time cost per individual, which is of course zero. *GC* denotes aggregate garage costs, so that $\frac{GC}{D}$ denotes mean (per individual) garage costs. Aggregate parking time per ml^2-hr equals the mean time parked $\left(\frac{1}{\gamma} = 2.0\right)$ times the rate at which parking spaces are filled per ml^2-hr (D = 7424). The ratio of time parked in parking garages to total time parked equals the ratio of garage parking spaces to total parking spaces $\left(\frac{\left(\frac{D}{\gamma} - P\right)}{\frac{D}{\gamma}} = 0.75\right)$. Thus, the mean time parked in a parking garage is 1.5 hrs, and since the resource cost of a garage space is $3.00/hr the mean resource cost associated with garage parking is $4.50. With no cruising for parking, the equilibrium travel speed[19] is 14.99 mph. Mean in-transit travel time cost, $\frac{TT}{D}$, is $3.05, calculated as travel time (trip length times travel time per mile, or trip length divided by velocity) multiplied by the mean value of travel time, $22.88. Aggregate resource costs equal aggregate garage costs plus aggregate in-transit travel time cost and aggregate cruising-for-parking time cost. Mean resource cost is therefore $7.55. The cells corresponding to

[19] Solve (1) with C = 0 and P = 3712 for T. Take the smaller root, T', which corresponds to the stable equilibrium, calculate travel time as $t(T', 0, P)$ and velocity as the reciprocal of travel time.

the visit length of the marginal parker for the various percentile values of time are blank, since the allocation of parkers between curbside and garage parking does not affect resource costs and is therefore indeterminate. And full trip prices are left blank because they do not pertain to social optimal allocations.

Column 2 records values associated with the full social optimum, where the amount of curbside parking is set optimally rather than at its base case level. The optimal number of curbside parking spaces is 4594 per ml^2-hr. Since this exceeds the amount of curbside parking in the base case social optimum, mean garage cost is lower in the full social optimum than in the base case social optimum but, since more road space is allocated to curbside parking, mean in-transit travel time cost is higher. Mean resource cost is $7.45. Thus, optimizing the amount of curbside allocated to parking reduces resource costs by only about ten cents per trip.

Column 3 records values for the base-case equilibrium in which curbside parking is on one side of the street. We explore this case in some detail. The stock of cars in transit and cruising for parking are determined simultaneously by two equilibrium conditions. The first is (1), the steady-state equilibrium condition, that the exit rate from the in-transit pool, $\frac{T}{\delta t}$, equal the exogenous entry rate D. The second, that the stock of cars cruising for parking be such that in equilibrium one-quarter of the aggregate time parked is curbside and the remainder in parking garages, is given by eqs. (12) – (14). The equilibrium values of T and C are $T = 1391.2$ and $C = 302.1$, so that 17.8% of cars are cruising for parking.

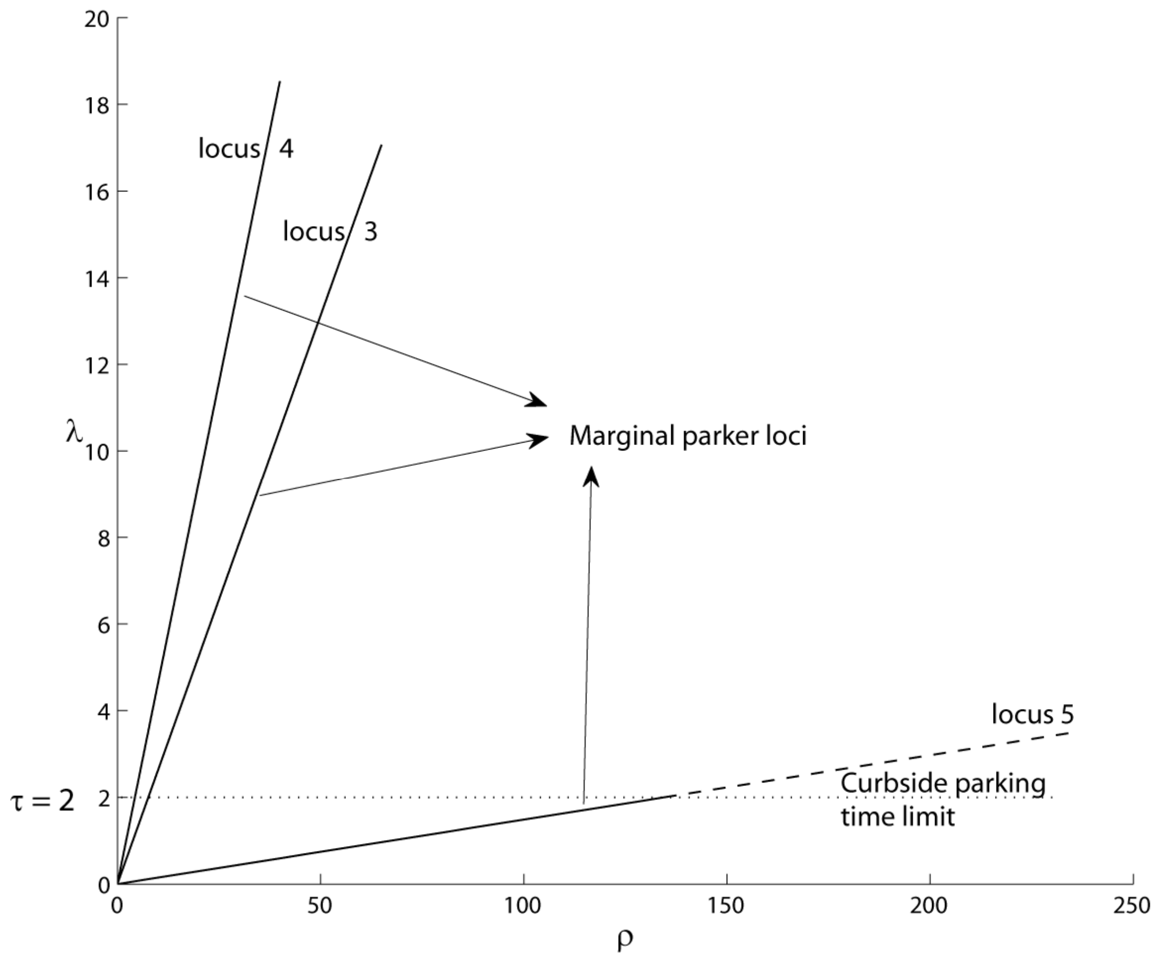
Recall that a marginal parker is one who is indifferent between curbside and garage parking. Figure 1 plots marginal parker loci in ρ-λ space. The marginal parker locus for the base case equilibrium (column 3) is labeled locus 3. For each value of time, those with the longest parking times choose to park curbside since they derive the greatest money saving from curbside rather than garage parking, enough to offset the fixed cost associated with cruising for parking. Thus, individuals above the marginal parker locus park curbside, while those below it park in a garage. Furthermore, since the fixed cost associated with cruising for parking is linearly proportional to the value of time, the length of time parked for a marginal driver is inversely related to her value of time. Consequently, the marginal parker locus is a ray from the origin with slope $\frac{C}{(c-f)\psi}$. The curbside parking turnover rate is 575.5. Since the number of curbside parking spaces is 3712, the average rate at which a parking space turns over is 0.155 times per hour, implying an average time parked of 6.45 hrs. The parking duration of the marginal parker with the 10^{th} percentile value of time ($\chi(10)$) is 3.56 hrs; with the median value of time, 5.63 hrs; and with the 90^{th} percentile value of time, 8.92 hrs. Compared to the corresponding social optimum shown in column 1: Mean garage cost is unchanged at \$4.50 since it depends only on the number of garage parking spaces; the price differential between curbside and garage parking generates cruising for parking, and mean cruising-for-parking time cost is \$0.69; cars cruising for parking increase traffic congestion, with travel speed falling to 10.67 mph and mean in-transit travel time cost increasing to \$4.29; and mean resource cost increases to \$9.48. Thus, the efficiency loss due to the underpricing of curbside parking is \$1.93 per individual.

An individual's full trip price increases with her value of time and visit duration. Consider for example, the individual with the median value of time, \$21.46/hr, and the median visit duration, 1.386 hrs. She parks in a garage, paying the garage fee of \$3.00/hr, so that her garage parking cost is \$4.16. Since she parks in a garage, she incurs no cruising-for-parking time cost. In-transit travel time equals travel distance divided by travel speed, 2.0/10.67 = 0.187 hrs so that her in-transit travel cost is \$4.02, and her full trip price \$8.18. Now consider the individual with the 10th percentile value of time of \$13.56/hr and 90th percentile visit duration of 4.605 hrs. She parks curbside. Her curbside parking costs are \$4.61; her expected time cruising for parking equals the stock of cars cruising for parking divided by the curbside parking turnover rate $\left(\frac{302.1}{575.5} = 0.525\ hrs.\right)$, so that her expected cruising-for-parking time cost is \$7.12. Her in-transit travel time cost is \$2.54, and her full trip price is \$14.27.

Column 4 gives equilibrium values for a policy environment differing from the base case equilibrium's only in that the amount of curbside parking is optimized. We first compare the results of column 4, the equilibrium with the second-best level of curbside parking, to those of column 2, the social optimum with the first-best level of parking. When individuals are identical, when curbside parking is underpriced, and when garage parking is priced at marginal cost, Arnott and Rowse (2009) proved that the second-best level of curbside parking falls short of the first-best level. The intuition is that the marginal cost of curbside parking is higher when curbside parking is underpriced due to the increased cruising for parking an extra curbside parking space induces, but the marginal benefit is

Figure 1: Marginal Parker Loci

Note: Marginal parker loci are numbered according to the corresponding column in Table 1. Above a marginal parker locus, an individual would prefer to park curbside; below it, he would prefer to park in a garage. The curbside parking time limit applies only to locus/column 5. In that case, for $\lambda > \tau = 2.0$ the curbside parking time limit binds so that individuals with visit duration greater than 2.0 must park in a garage; in this region, the marginal parker locus does not determine whether an individual parks curbside or in a garage. The marginal parker locus for column 5 is drawn as a solid line below $\lambda = 2.0$ and as a dashed line above it (where the parking time limit applies).



unchanged (simply equaling the savings in garage parking cost). The same result can be shown to hold when individuals are heterogeneous. Thus, the optimal amount of curbside parking is lower in the equilibrium (1044 parking spaces per ml^2) than in the corresponding social optimum (4594 parking spaces per ml^2).

Now compare column 4's equilibrium, which has the second-best optimal amount of curbside parking, to column 3's equilibrium, which has the base case amount of curbside parking (3712 parking spaces per ml^2). Since the second-best amount of curbside parking is lower than the base-case amount, mean garage cost is higher in the second-best equilibrium, but this is more than offset by the decrease in mean travel time cost, so that mean resource cost falls by $1.17 compared to the base case equilibrium. Furthermore, the reduction in the amount of curbside parking decreases the stock of cars cruising for parking, and via the steady-state equilibrium condition the stock of cars in transit as well. Garage parkers benefit from the reduction in congestion, and curbside parkers from the reduction in both congestion and cruising-for-parking time.

How the equilibrium changes with the reduction in the amount of curbside parking corresponds to the change in the solution to eqs. (12) – (14) as P is lowered, which is analyzed in Appendix 1. Turn to Figure 1. Recall that the marginal parker locus is a ray from the origin, and that those above the locus choose to park curbside. The reduction in the amount of curbside parking therefore causes the marginal parker locus to rotate counterclockwise from locus 3 to locus 4; by how much depends on the density of parkers between locus 3 and locus 4. The rotation of the locus causes the marginal parker for each visit length to have a lower value of time, and it can be shown that the mean value of time of curbside parkers, $\overline{\rho}$, and also of marginal curbside parkers,

$\bar{\tilde{\rho}}$, unambiguously fall[20]. The rotation of the locus also causes the marginal parker to have a longer visit duration for each value of time, and it can be shown that the average visit duration of curbside parkers, $\bar{\lambda}$, and also of marginal curbside parkers, $\bar{\tilde{\lambda}}$, unambiguously rise. The curbside turnover rate, ψ, falls proportionally more than the amount of curbside parking since average visit length increases. It follows from (14) that the stock of cars cruising for parking is[21]

$$C = (c-f)\psi\left(\frac{\bar{\tilde{\lambda}}}{\bar{\tilde{\rho}}}\right) = \frac{(c-f)\mathrm{P}\left(\frac{\bar{\tilde{\lambda}}}{\bar{\tilde{\rho}}}\right)}{\bar{\lambda}}. \qquad (17)$$

In the example: The slope of the marginal parker locus increases from 0.263 to 0.463; expected cruising-for-parking time increases by the same proportional amount, from 0.525 hrs to 0.926 hrs; the stock of cars cruising for parking falls somewhat less than proportionally to the reduction in P, from 302.14 to 110.61; the turnover rate decreases more than proportionally to P, from 575.5 to 119.4; the mean visit length of curbside parkers increases from 6.45 hrs to 8.74 hrs; the mean value of time of curbside parkers

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[20] These results, and the corresponding results for $\bar{\lambda}$ and $\bar{\tilde{\lambda}}$, hold when the value of time and visit duration are independently distributed, which the example assumes, but not generally otherwise. See the Appendix.

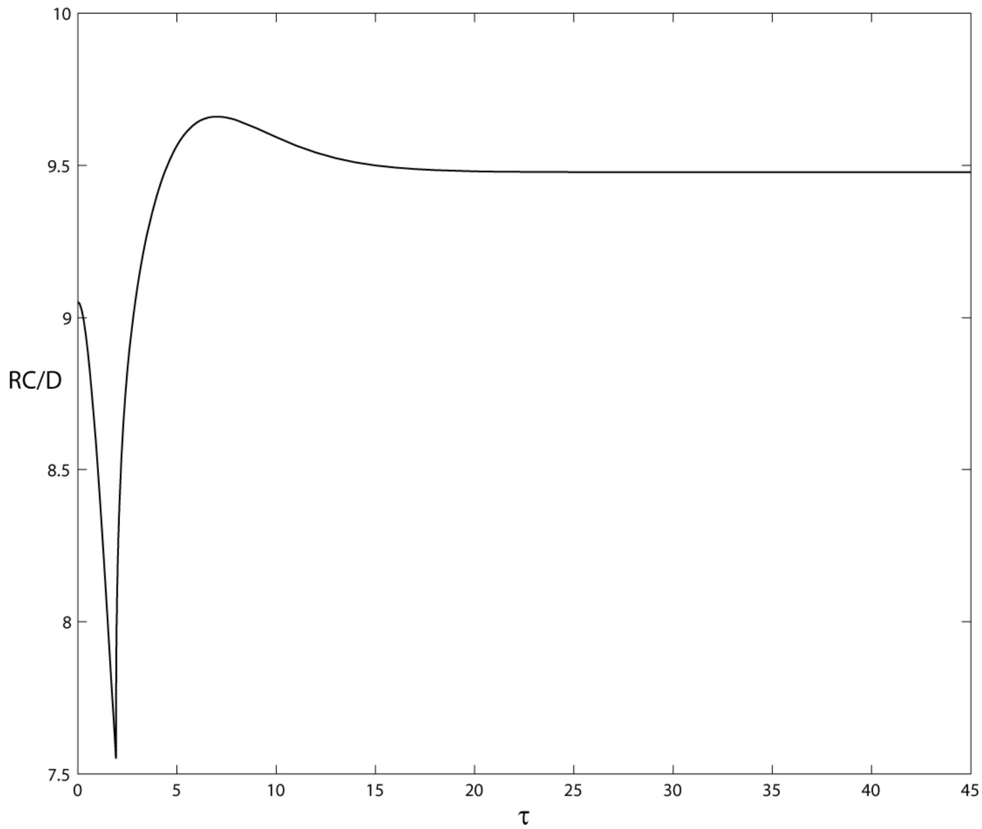
[21] Recalling that $\frac{\bar{\tilde{\lambda}}}{\bar{\tilde{\rho}}}$ is the slope of the marginal parker locus, the elasticity of the stock of cars cruising for parking with respect to *P* equals one plus the elasticity of the slope of the marginal parker locus with respect to *P* minus the elasticity of the mean visit length of curbside parkers with respect to *P*. Mean cruising-for-parking cost equals expected cruising-for-parking time (which equals the stock of cars cruising for parking divided by turnover rate and is proportional to the slope of the marginal parker locus) times the mean value of time of curbside parkers. Thus, the elasticity of mean cruising-for-parking cost with respect to *P* equals the elasticity of the slope of the marginal parker locus with respect to *P* plus the elasticity of the mean value of time of curbside parkers with respect to *P*.

decreases from $16.95/hr to $14.56/hr; and mean cruising-for-parking cost falls from $0.690 to $0.217.

Column 5 gives equilibrium values for a policy environment differing from the base case equilibrium's only in that a curbside parking limit of two hours is introduced. The effects of this curbside parking time limit are seen by comparing Column 5 with column 3. The immediate effect of the time limit is to force those with visit durations exceeding two hours to garage park. Since one quarter of the aggregate time parked is still curbside, equilibrium requires that the marginal parker locus, labeled locus 5 in Figure 1, rotate clockwise. It is shown as a dashed line for visit length exceeding two hours, where the time limit binds. In the Figure, curbside parkers occupy the triangle above the marginal parker locus and below $\lambda = 2$, with garage parkers occupying the rest of the space. The overall effect of the curbside parking time limit is that the mean visit length for curbside parkers falls sharply; the rise in the curbside parking turnover rate from 575.5 parking spaces per ml^2-hr to 3542.94 corresponds to a reduction in mean visit length of curbside parkers from 6.45 hrs to slightly over one hour. The mean value of time of curbside parkers rises somewhat, the outcome of two offsetting effects; on one hand, no individuals with very high values of time find it worthwhile to cruise for parking when visit length is restricted to two hours, on the other the mean value of time of curbside parkers with visit lengths of less than two hours increases. The stock of cars cruising for parking is determined by the characteristics of *marginal* parkers; see (17). Even though the turnover rate increases sixfold, the stock of cars cruising for parking falls by a factor of almost three since the ratio of visit duration to value of time of marginal parkers (the slope of the marginal parker locus) falls by a factor of almost eighteen. The curbside parking limit has no effect on mean garage cost but causes a reduction in mean cruising-for-parking cost from $0.69 to $0.31 and in mean in-transit travel cost from $4.29 to $3.33 (corresponding to an increase in traffic speed from 10.67 mph to 13.72 mph.), resulting in a mean resource saving of $1.33. The curbside parking time limit does not, however, benefit all groups. Consider first individuals with a zero value of time. The full trip price of those with visit durations of less than two hours is unaffected by the time limit, while the full trip price of those with visit durations exceeding two hours increases by the amount $(c - f)\lambda$ since they have to pay the garage rather than the curbside parking fee. Note also that no individual with a visit duration of less than two hours, or who garage parked before the introduction of the curbside parking time limit, is hurt by the policy. Thus, there is a locus in the region where visit duration exceeds two hours, to the left of which individuals are hurt by the policy and to right of which individuals are helped. The table reports the full trip price of nine individuals. The only one who is hurt by the policy has the 10^{th} percentile value of time and the 90^{th} percentile visit duration.

Column 6 considers the same policy environment as column 5 except that the curbside parking time limit is optimized. As was proved earlier, the optimal curbside parking time limit causes the visit duration of all marginal parkers to fall to zero, so that the marginal parker locus coincides with the ρ-axis, and results in the demand for curbside parking equaling the supply, with no cruising for parking. Comparing columns 1 and 6 confirms that the optimal curbside parking limit decentralizes the base case social optimum.

Figure 2: Mean resource cost as a function of the curbside parking time limit



The optimal curbside parking time limit is 1.92 hrs, which contrasts with the previous case where the time limit was 2.0 hrs. The most striking effect of the reduction in the curbside parking limit from 2.0 to 1.92 hrs is by how much mean resource cost falls – from $8.14 to $7.56 – given that the policy change is so small. Why? In Figure 2, which plots mean resource cost against the curbside parking time limit, the optimal curbside parking time limit is not a regular minimum. Rather, there is a slope discontinuity in mean resource cost at the minimum. If the parking time limit is set below the optimum, there is an excess supply of curbside parking spaces (and no cruising for parking) and, to accommodate the demand for parking, extra garage spaces have to be built, using up resources[22]. This does not, however, explain why aggregate resource costs fall so rapidly as the time limit is lowered from 2.0 to 1.93 hrs. To investigate this outcome, consider the simpler situation where individuals have the same value of time. If the stock of cars cruising for parking falls very rapidly as the curbside parking time limit falls towards its optimal level, so too will resource costs. From (10b):

$$E_{C:\tau} = E_{\tilde{\lambda}:\tau} - E_{\Lambda:\tau}\ , \tag{18}$$

where Λ is the mean visit length of curbside parkers. In the limit, as the curbside parking time limit is reduced to its optimal level, the visit length of the marginal parker falls at a proportionally infinite rate while the mean visit length of curbside parkers falls at a proportionally finite rate.

Another interesting feature of Figure 2 is that resource cost per capita is *maximized* not in the absence of curbside parking time limits but with a time limit of about six hours. Put alternatively, the rationing mechanism for curbside parking with a time limit of six hours is more inefficient than the rationing mechanism with no time limit. This result too is easier to explain when individuals have the same value of time. In that case, resource costs per capita as a function of τ can be written as

[22] When the curbside parking time limit is set below the optimum, mean resource cost is straightforward to calculate. Since there is no cruising for parking, mean cruising-for-parking cost is zero and mean in-transit travel cost the same as in the social optimum with P = 3712 (recall that the specification of the congestion technology assumes that the proportion of the street available for traffic circulation is based on the *designated* number of curbside parking spaces rather than the occupied number, on the argument that even one car parked curbside on a block precludes the curbside lane from being used for traffic circulation). Since all those with visit lengths shorter than the time limit park curbside, the number of garage spaces needed is $D\int_0^{\infty}\left[\int_{\tau}^{\infty}\lambda g(\rho,\lambda)d\lambda\right]d\rho$. In the limit, as the curbside parking time limit goes to zero, everyone garage parks so that $\frac{GC}{D}$ = 6.00 and $\frac{RC}{D}$ = 9.053.

$$\frac{RC(\tau)}{D} = \frac{\rho T\big(C(\tau)\big)}{D} + \frac{\rho C(\tau)}{D} + \frac{GC}{D}, \tag{19}$$

where $T(C)$ is the level of T associated with a given level of C consistent with the steady-state condition, when the stability condition on the choice of root is imposed, and for which $T'(C) > 0$. Since garage costs per capita are independent of τ, a change in τ causes $\frac{RC}{D}$ and C to move in the same direction. Then explaining why C need not be monotonically increasing in τ also explains why $\frac{RC}{D}$ need not be monotonically increasing in τ. The sign of $\frac{dC}{d\tau}$, which is given by (11), is clearly positive when $\tilde{\lambda}$ is small, as it is for a curbside parking limit slightly above the optimal level, or with a uniform distribution of visit duration, but is negative if the probability density function of visit duration declines sufficiently rapidly.

This paper analyzes steady states. In reality, traffic varies systematically over the day and over locations, and is subject to stochastic fluctuations as well. To achieve the ideal of saturated parking with no cruising for parking, the curbside parking time limit would have to vary not only over location and over the day but also in response to realized parking demand, as advocated by Vickrey (1954) and Shoup (2006). While feasible given current technology, “fine” curbside parking time limits are rarely observed and instead parking time limits are typically fixed over the business day. That resource costs are sensitive to how close the curbside parking time limit is to the optimal level suggests that actual curbside parking time limits achieve only a fraction of their potential efficiency gains and at the same time that the efficiency gains to moving to finer curbside parking time limits may be significant.

Column 7 describes the equilibrium when the curbside parking time limit and the amount of curbside parking are simultaneously optimized. This equilibrium is identical to the corresponding social optimum, except that the allocation of individuals across curbside and garage parking is determinate[23].

Column 8 shows what the base case equilibrium would be if all individuals had a value of time and visit duration equal to their mean values. Moving from column 8 to column 3 therefore indicates how the equilibrium changes when the value of time and visit length become heterogeneous. The main result is that accounting for heterogeneity reduces calculated mean resource cost. Intuition might suggest that, when individuals are heterogeneous, they sort themselves systematically between curbside and garage parking in a way that *always* lowers mean resource cost -- this is the "value pricing" effect[24]. This sorting lowers mean resource cost in the example, but not always. Consider a situation with two groups of individuals, one with a very low value of time, the other with a very high value of time, and suppose that the amount of curbside allocated to parking is

[23] Interestingly, for all the nine individuals reported on, the full trip price in this equilibrium is higher than in the equilibrium when only the curbside parking time limit is chosen optimally. Because the first-best amount of curbside parking exceeds the base case level, more revenue is raised from curbside parking in the equilibrium when both the curbside parking time limit and the amount of curbside parking are optimized than when only the curbside parking time limit is optimized. Such revenue is a transfer rather than a social cost.

[24] Suppose that there are two congestible roads linking an origin and a destination, that individuals are identical, and that demand is sensitive to price. Numerical examples in earlier literature found that only a small proportion of the potential efficiency gains from congestion tolling are achieved when a (second-best) toll can be applied to only one of the roads. Small and Yan (2001) showed that the proportion of the potential efficiency gains is considerably higher when individual heterogeneity is taken into account; those with high values of time choose the tolled road, those with low values of time the untolled road. They termed this phenomenon "the value of value pricing".

such that the marginal parker has a very low value of time. The stock of cars cruising for parking needs to be high for him to be indifferent between curbside and garage parking, but this causes the in-transit travel costs of those with the very high value of time to be very large.

Finally, we contrast the equilibria in which the base case optimum is decentralized through raising the curbside parking fee to the efficient level ($f = 3$) rather than through imposing the optimal curbside parking time limit. We refer to the former as the efficient curbside pricing equilibrium and the latter as the efficient time limit equilibrium. The two equilibria have identical resource costs but different distributional effects. In particular, while the full trip prices of garage parkers are the same in both equilibria, those of inframarginal curbside parkers are lower with the curbside parking time limit. More curbside parking revenue is raised under the efficient pricing equilibrium. If this increase in revenue is uniformly redistributed, then moving from the efficient time limit equilibrium to the efficient curbside pricing equilibrium hurts curbside parkers but helps garage parkers.

5. Concluding Comments

This paper presented a very simple model of downtown parking in order to provide a partial rationale for curbside parking time limits. It took as given that the curbside meter rate is set below the garage parking fee, which is set equal to the constant marginal cost of a garage parking space. This provides an incentive for individuals to cruise for the cheaper, curbside parking, leading not only to cruising-for-parking costs but also increased traffic congestion. It considered a situation where individuals differ in terms of both value of time and visit length. When there is cruising for parking, individuals sort themselves between curbside and garage parking according to these characteristics. In particular, those with higher values of time choose garage parking since they value the time savings from not having to cruise for parking more highly, while those with longer visit durations choose curbside parking since they receive a larger subsidy from curbside parking.

The principal contribution of the paper was to show that in this situation a curbside parking time limit that sets a maximum length of stay at a curbside parking spot can be used to eliminate the cruising for parking generated by the price differential between curbside and garage parking. We provided two complementary explanations of how curbside parking limits do this.

Of course, the world is not as simple as our model: the demand for curbside parking varies over the business day, while curbside parking time limits generally do not; parking enforcement is costly and illegal parking may be important; downtown is not spatially uniform; garages are discretely spaced, which confers market power on garage owners[25]; the demand for trips downtown is not inelastic; and the distribution of visit lengths is not

[25] When parking garages price above marginal cost, pricing curbside parking to eliminate excess demand entails pricing above garage marginal cost too. Thus, a second-best argument can be made for pricing curbside parking lower than garage parking and using curbside parking time limits to eliminate the excess demand.

exogenous[26]. Consideration of these factors, while important, should not undermine the basic insights derived from our model.

[26] The distribution of trip durations can be endogenized by assuming that there is a distribution of trip benefit functions, each of which specifies the trip benefit as a function of visit duration, as is done in Glazer and Niskanen (1992). Individuals then decide on the duration of a visit to maximize net trip benefit. But even this specification is not completely satisfactory since it ignores activity rescheduling, trip chaining, and the choice between parking once and walking between destinations, or driving between destinations and parking at each.

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Notational Glossary

α	slope of the marginal parker locus $\left(\frac{\tilde{\lambda}}{\tilde{\rho}}\right)$
γ	parameter of negative exponential visit time distribution
δ	driving distance
$\varepsilon(\rho)$	visit duration of marginal parker with value of time ρ
λ	visit duration
$\bar{\lambda}$	mean visit duration of curbside parkers
$\bar{\tilde{\lambda}}$	mean visit duration of marginal parkers
μ	mean of ln ρ
θ	passenger car equivalent (in terms of effective density) of a car cruising for parking
ρ	value of time
$\bar{\rho}$	mean value of time of curbside parkers
$\tilde{\rho}$	value of time of marginal parker
$\bar{\tilde{\rho}}$	mean value of time of marginal parkers
$\xi(\alpha)$	intermediate function
σ	standard deviation of ln ρ
τ	curbside maximum parking time limit
$\chi(\%)$	visit length of marginal parker with percentile value of time $\%$
ψ	turnover rate of curbside parking
$\Lambda(\lambda')$	mean visit length for $\lambda \geq \lambda'$
$\Lambda(\lambda',\lambda'')$	mean visit length for $\lambda \in (\lambda', \lambda'')$
Ω	effective jam density with no curbside parking
c	unit garage parking cost per unit time
f	curbside meter rate per unit time
$g(\rho, \lambda)$	joint p.d.f. of value of time and visit duration
$h(\lambda)$	p.d.f. of visit duration
$j(\rho)$	p.d.f. of value of time

t	travel time per mile
t_0	free flow travel time
v	velocity
$w(\rho;\alpha)$	p.d.f of value of time of curbside parkers
$x(\rho;\alpha)$	p.d.f. of value of time of marginal parkers
$y(\lambda;\alpha)$	p.d.f. of visit duration of curbside parkers
z	intermediate function
C	stock of cars cruising for parking per unit area-time
CP	cruising-for-parking costs per unit area-time
D	trip demand per unit area-time
$E_{x:y}$	elasticity of x with respect to y
$F(\%,\%')$	full price for the parker with the $\%$ percentile value of time and the $\%'$ percentile visit duration
$G(\rho,\lambda)$	c.d.f. of value of time and visit duration
GC	garage costs per unit area-time
$H(\lambda)$	c.d.f. of visit length
$J(\rho)$	c.d.f. of value of time
P	stock of curbside parking spaces per unit area-time
P_{max}	maximum possible stock of curbside parking spaces per unit area-time
RC	resource costs per unit area-time
SO	social optimum
T	stock of cars in transit per unit area-time
V	effective density per unit area-time
V_j	effective jam density
$W(\rho;\alpha)$	c.d.f of value of time of curbside parkers
$Y(\lambda;\alpha)$	c.d.f. of visit duration of curbside parkers

Appendix

The Effects of an Increase in the Amount of Curbside Parking

Defining $\alpha = \frac{\varepsilon(\rho)}{\rho}$, which is the slope of the marginal parker locus, (12) – (14) can be rewritten as

$$D\int_0^\infty \left[\int_{\rho\alpha}^\infty \lambda g(\rho,\lambda)d\lambda\right]d\rho = P \tag{A1}$$

$$\psi = D\int_0^\infty \left[\int_{\rho\alpha}^\infty g(\rho,\lambda)d\lambda\right]d\rho \tag{A2}$$

$$C = (c-f)\alpha\psi \tag{A3}$$

The first equation states that the demand for curbside parking equals the supply; the second indicates that the curbside parking turnover rate equals the inflow rate into curbside parking; and the third gives the equilibrium stock of cars cruising for parking.

Taking the ratio of (A1) to (A2) gives

$$\bar{\lambda}(\alpha)\psi = \mathrm{P}, \tag{A4}$$

where

$$\bar{\lambda}(\alpha) = \frac{\int_0^\infty \left[\int_{\rho\alpha}^\infty \lambda g(\rho,\lambda)d\lambda\right]d\rho}{\int_0^\infty \left[\int_{\rho\alpha}^\infty g(\rho,\lambda)d\lambda\right]d\rho}. \tag{A5}$$

Eq. (A4) says simply that the turnover rate times the average time parked curbside equals the stock of curbside parking spaces

Totally differentiating (A1) and (A2) yields

$$-D\left[\frac{d\alpha}{dP}\right]\int_{0}^{\infty}[\alpha\rho]\rho g(\rho,\alpha\rho)d\rho = 1 \tag{A6}$$

and

$$-D\left[\frac{d\alpha}{dP}\right]\int_{0}^{\infty}\rho g(\rho,\alpha\rho)d\rho = \frac{d\psi}{dP}\,. \tag{A7}$$

Let $\bar{\tilde{\lambda}}(\alpha)$ denote the mean visit duration of marginal parkers. Since $\frac{\rho g(\rho,\alpha\rho)}{\int_0^{\infty}\rho g(\rho,\alpha\rho)d\rho}$ is the density function of marginal curbside parkers, and since $\varepsilon(\rho) = \rho\alpha$ is the visit duration of the marginal parker with value of time ρ:

$$\bar{\tilde{\lambda}}(\alpha) = \frac{\int_0^{\infty}[\alpha\rho]\rho g(\rho,\alpha\rho)d\rho}{\int_0^{\infty}\rho g(\rho,\alpha\rho)d\rho}. \tag{A8}$$

Taking the ratio of (A6) to (A7) and using (A8) yields

$$\bar{\tilde{\lambda}}(\alpha)\left[\frac{d\psi}{dP}\right] = 1. \tag{A9}$$

Combining (A4) and (A9) gives

$$E_{\psi:P} = \frac{\bar{\lambda}}{\bar{\tilde{\lambda}}(\alpha)}\,, \tag{A10}$$

where $E_{x:y}$ denotes the elasticity of x with respect to y. We may rewrite (A4) in elasticity form as

$$E_{\bar{\lambda}:P} + E_{\psi:P} = 1\,. \tag{A11}$$

From (A3):

$$E_{C:P} = E_{\alpha:P} + E_{\psi:P} \,. \tag{A12}$$

From (A1) and (A6), we obtain[27]

$$E_{\alpha:P} = -\frac{\int_0^\infty \left[\int_{\rho\alpha}^\infty \lambda g(\rho,\lambda)d\lambda\right] d\rho}{\int_0^\infty [\alpha^2\rho]\rho g(\rho,\alpha\rho)d\rho} < 0 \,, \tag{A13}$$

which is the percentage change in the slope of the marginal parker locus induced by a percentage change in the stock of curbside parking spaces. (A12) and (A13) together imply that $E_{C:P} < E_{\psi:P}$. The mean value of time of curbside parkers is

$$\overline{\rho}(\alpha) = \frac{\int_0^\infty \left[\int_{\rho\alpha}^\infty \rho g(\rho,\lambda)d\lambda\right] d\rho}{\int_0^\infty \left[\int_{\rho\alpha}^\infty g(\rho,\lambda)d\lambda\right] d\rho}, \tag{A14}$$

while the mean value of time of marginal parkers is

$$\overline{\tilde{\rho}}(\alpha) = \frac{\overline{\tilde{\lambda}}(\alpha)}{\alpha} \tag{A15}$$

(which implies that $1 + E_{\overline{\tilde{\rho}}:\alpha} = E_{\overline{\tilde{\lambda}}:\alpha}$ and that $E_{\alpha:P} + E_{\overline{\tilde{\rho}}:P} = E_{\overline{\tilde{\lambda}}:P}$). Expected cruising-for-parking time is $\frac{C}{\psi}$, which is directly proportional to α, and mean cruising-for-parking cost is $\frac{\overline{\rho}C}{D}$ (so that the elasticity of mean cruising-for-parking cost with respect to P equals $E_{\overline{\rho}:P} + E_{C:P}$).

Observe that all the above means and elasticities depend only on the properties of $g(\rho,\lambda)$.

[27] This equation looks strange. A more elegant formulation would use polar coordinates, in which case the analog of (A13) would give the elasticity of parking demand with respect to $\frac{\pi}{2}$ minus the angle subtended by the marginal parker locus.

It is true in general that α and P are negatively related, and that an increase in P causes the ratio of visit length to value of time for marginal parkers, as well as the mean ratio of visit length to the value of time for all curbside parkers, to fall. But beyond these results, little can be said in general. One might think that the mean visit length of marginal curbside parkers is less than the mean visit length of inframarginal curbside parkers, for example, but this is incorrect. Marginal parkers have the lowest *ratio* of visit length to value of time among all curbside parkers. If, however, marginal parkers have a mean value of time that is considerably higher than the mean for all curbside parkers, they may have a higher mean visit length than inframarginal curbside parkers.

To investigate further the theoretical basis for our numerical results, we shall impose the assumption that value of time and trip duration are independently distributed: $g(\rho,\lambda) = j(\rho)h(\lambda)$, where $j(.)$ is the marginal distribution of the value of time and $h(.)$ the marginal distribution of visit length, and see what additional results we obtain. We term this assumption A-1.

Result 1: Under A-1, $\frac{d\bar{\rho}}{dP} > 0$

Proof: The proof is based on a standard first-order stochastic dominance argument. We know that $\frac{d\alpha}{dP} < 0$. Thus, it suffices to show that $\overline{\rho}'(\alpha) < 0$. Since the joint density function is the product of the marginal density functions, the expression for $\bar{\rho}$ in (A14) reduces to

$$\bar{\rho}(\alpha) = \frac{\int_0^\infty \rho j(\rho)\big(1 - H(\alpha\rho)\big)d\rho}{\int_0^\infty j(\rho)\big(1 - H(\alpha\rho)\big)d\rho}, \qquad \text{(A16)}$$

which may be rewritten as

$$\overline{\rho}(\alpha) = \int_0^\infty \rho w(\rho;\alpha)d\rho, \tag{A17}$$

$$where\ w(\rho;\alpha) = \frac{j(\rho)\big(1 - H(\alpha\rho)\big)}{\int_0^\infty j(\rho)(1 - H(\alpha\rho))d\rho}.$$

Integrating (A17) by parts yields

$$\overline{\rho}(\alpha) = [\rho W(\rho;\alpha)]_0^\infty - \int_0^\infty W(\rho;\alpha)d\rho. \tag{A18}$$

Differentiating (A18) with respect to α gives

$$\overline{\rho'}(\alpha) = [\rho W_\alpha(\rho;\alpha)]_0^\infty - \int_0^\infty W_\alpha(\rho;\alpha)d\rho. \tag{A19}$$

Since an increase in α compresses the p.d.f. towards the origin, $W_\alpha(\rho,\alpha) > 0$ for $\rho \in (0,\infty)$ (proof available upon request). Also, since $W(.)$ is a c.d.f., $W(0;\alpha) = 0$, $W_\alpha(0;\alpha) = 0$, $W(\infty;\alpha) = 1$, and $W_\alpha(\infty;\alpha) = 0$. As well, under reasonable regularity conditions (which are satisfied in the example), $[\rho W_\alpha(\rho;\alpha)]_0^\infty = 0$. The result then follows from (A19). □

Corollary 1: Under A-1, $\overline{\rho}(\alpha) < \overline{\overline{\rho}}(\alpha)$

Proof: Since the average is falling in α, and since the average equals the marginal at α = 0, the marginal is less than the average. □

Result 2: Under A-1, $\frac{d\overline{\lambda}}{dP} < 0$.

Proof: The proof is analogous to that for Result 1. With the order of integration reversed,

$$\overline{\lambda}(\alpha) = \frac{\int_0^\infty \lambda \left[\int_{\frac{\lambda}{\alpha}}^\infty j(\rho)d\rho\right] h(\lambda)d\lambda}{\int_0^\infty \left[\int_{\frac{\lambda}{\alpha}}^\infty j(\rho)d\rho\right] h(\lambda)d\lambda} = \int_0^\infty \lambda y(\lambda;\alpha)d\lambda, \tag{A20}$$

$$where\ y(\lambda;\alpha) = \frac{h(\lambda)\left(1 - J\left(\frac{\lambda}{\alpha}\right)\right)}{\int_0^\infty h(\lambda)\left(1 - J\left(\frac{\lambda}{\alpha}\right)\right)d\lambda}.$$

Integrating (A20) by parts yields

$$\overline{\lambda}(\alpha) = [\lambda Y(\lambda;\alpha)]_0^\infty - \int_0^\infty Y(\lambda;\alpha)d\lambda. \tag{A21}$$

Differentiate (A20) with respect to α:

$$\overline{\lambda'}(\alpha) = [\lambda Y_\alpha(\lambda;\alpha)]_0^\infty - \int_0^\infty Y_\alpha(\lambda;\alpha)d\lambda. \tag{A22}$$

The argument from here is completely analogous to that for Result 1, except that an increase in α stretches the p.d.f. away from the origin, with the result that $Y_\alpha(\lambda;\alpha) < 0$ for $\lambda \in (0,\infty)$. The result then follows from (A22) and $\frac{d\alpha}{dP} < 0$. □

Corollary 2: Under A-1, $\overline{\lambda}(\alpha) > \overline{\overline{\lambda}}(\alpha)$

Proof: Since the average is increasing in α, and since the average equals the marginal at $\alpha = 0$, the marginal is greater than the average. □

Result 3: Under A-1: i) $E_{\psi:P} > 1$; ii) $E_{\overline{\lambda}:P} < 0$; iii) $E_{\overline{\rho}:P} > 0$; iv) $E_{\overline{\overline{\rho}}:P} > 0$; and v) $E_{\overline{\overline{\lambda}}:P} < 0$.

Proof: ii) follows directly from Result 2 and iii) directly from Result 1. i) follows from ii) and (A11).

$$\overline{\overline{\rho}}(\alpha) = \frac{\int_0^\infty [\rho]\rho j(\rho)h(\alpha\rho)d\rho}{\int_0^\infty \rho j(\rho)h(\alpha\rho)d\rho} = \int_0^\infty \rho x(\rho;\alpha)d\rho, where\ x(\rho;a) = \frac{\rho j(\rho)h(\alpha\rho)}{\int_0^\infty \rho j(\rho)h(\alpha\rho)d\rho}.$$

An increase in α transfers probability weight to the left in the distribution. By a now familiar, first-order stochastic dominance argument, this implies that $\overline{\tilde{\rho}'}(\alpha) < 0$, which along with $\frac{d\alpha}{dP}$ gives result iv). v) is obtained by reversing the order of integration, and applying an analogous argument. □

Note that the results thus far tell us little about one of the elasticities of central interest, $E_{C:P}$. With identical individuals, this elasticity equals one. With heterogeneous individuals, we have $E_{C:P} = E_{\psi:P} + E_{\alpha:P}$. From (A13):

$$E_{\alpha:P} = -\frac{\int_0^\infty \left[\int_{\rho\alpha}^\infty \lambda g(\rho,\lambda)d\lambda\right] d\rho}{\int_0^\infty [\alpha^2\rho]\rho g(\rho,\alpha\rho)d\rho} \tag{A23}$$

$$= -\frac{\overline{\lambda}(\alpha)}{\overline{\overline{\lambda}}(\alpha)}\left\{\frac{\int_0^\infty \left[\int_{\rho\alpha}^\infty g(\rho,\lambda)d\lambda\right] d\rho}{\int_0^\infty \rho g(\rho,\alpha\rho)d\rho}\right\}.$$

Under A-1, this reduces to

$$E_{\alpha:P} = -\frac{\overline{\lambda}(\alpha)}{\overline{\overline{\lambda}}(\alpha)}\left\{\frac{\int_0^\infty j(\rho)(1-H(\alpha\rho))\, d\rho}{\int_0^\infty \rho j(\rho)h(\alpha\rho)d\rho}\right\}. \tag{A24}$$

Now define[28]

[28] To illustrate the calculation of $\xi(\alpha)$, $\overline{\lambda}(\alpha)$, and $\overline{\overline{\lambda}}(\alpha)$, suppose that $j(\rho)$ and $h(\lambda)$ are both negative exponential distributed, the former with exponent β and the latter with exponent γ. Then $h(\alpha\rho) = \gamma e^{-\gamma\alpha\rho}$ and $\left(1-H(\alpha\rho)\right) = e^{-\gamma\alpha\rho}$. Thus, $\int_0^\infty j(\rho)\left(1-H(\alpha\rho)\right) d\rho = \int_0^\infty \beta e^{-(\beta+\gamma\alpha)\rho} d\rho = \frac{\beta}{\beta+\gamma\alpha}$ and $\int_0^\infty \rho j(\rho)h(\alpha\rho)d\rho = \int_0^\infty \rho\beta\gamma e^{-(\beta+\gamma\alpha)\rho}d\rho = \frac{\beta\gamma}{(\beta+\gamma\alpha)^2}$, so that $\xi(\alpha) = \frac{\beta+\gamma\alpha}{\gamma}$. Also, $\int_0^\infty \left[\int_{\rho\alpha}^\infty \lambda g(\rho,\lambda)d\lambda\right] d\rho = \int_0^\infty \left(\frac{\beta}{\gamma}\right)(1+\gamma\alpha\rho)e^{-(\beta+\gamma\alpha)\rho}d\rho = \frac{\beta}{\gamma(\beta+\gamma\alpha)} + \frac{\beta\alpha}{(\beta+\gamma\alpha)^2} = \frac{\beta(\beta+2\gamma\alpha)}{\gamma(\beta+\alpha)^2}$, so that $\overline{\lambda}(\alpha) = \frac{\beta+2\gamma\alpha}{\gamma(\beta+\gamma\alpha)}$ Also, $\int_0^\infty \alpha\rho^2 j(\rho)h(\alpha\rho)d\rho = \int_0^\infty \rho^2\alpha\beta\gamma e^{-(\beta+\gamma\alpha)\rho}d\rho = \frac{2\alpha\beta\gamma}{(\beta+\gamma\alpha)^3}$, so that $\overline{\overline{\lambda}}(\alpha) = \frac{2\alpha}{\beta+\gamma\alpha}$ Thus, $E_{\psi:P} = \frac{\beta+2\gamma\alpha}{2\gamma\alpha}$, $E_{\alpha:P} = -\frac{(\beta+2\gamma\alpha)(\beta+\gamma\alpha)}{2\gamma^2\alpha}$, and $E_{C:P} = \left(\frac{\beta+2\gamma\alpha}{2\gamma\alpha}\right)\left(1-\frac{\beta+\gamma\alpha}{\gamma}\right)$.

$$\xi(\alpha) = \left\{ \frac{\int_0^\infty j(\rho)\big(1 - H(\alpha\rho)\big)\, d\rho}{\int_0^\infty \rho j(\rho) h(\alpha\rho) d\rho} \right\}. \tag{A25}$$

Then we have

$$E_{\alpha:P} = -E_{\psi:P}\xi(\alpha)\,. \tag{A26}$$

so that from (A12)

$$E_{C:P} = \big(1 - \xi(\alpha)\big)E_{\psi:P} = \big(1 - \xi(\alpha)\big)\frac{\bar{\lambda}(\alpha)}{\bar{\bar{\lambda}}(\alpha)}. \tag{A27}$$